

Equivalence of states

Given $P, Q \in Q$, then

→ if $P = f \neq Q = f \Rightarrow P \neq Q$ are distinguishable and we can not merge them.

→ if (P, Q) are non-distinguishable, then they can be merged, means both should be final states or non-final states.

Example 1 (DFA Minimization)

DFA → Algo of ES → Minimized DFA

or optional minimized DFA

16 FRIDAY
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17 SATURDAY 18 SUNDAY
جمادی الاول ۲ جمادی الاول ۳

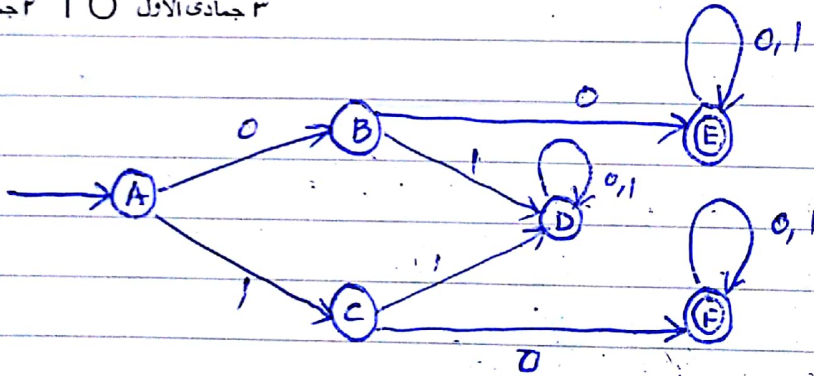


fig A

Then we build a $N \times N$ Step Table

A					
B					
C					
D					
E					
	A	B	C	D	E

Since

$$(B, D) \equiv (D, B)$$

$$(AA) \equiv A$$

So, we ^{don't} need ~~not~~

the crossed portion of the table

APRIL						
M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
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step

Hence the rest of the table becomes

B				
C				
D				
E				
	A	B	C	D

By applying the concept, the step table of fig A becomes

B	X ¹				
C	X ¹				
D	X ²	X ¹	X ¹		
E	X ⁰	X ⁰	X ⁰	X ⁰	
F	X ⁰	X ⁰	X ⁰	X ⁰	
	A	B	C	D	E

MONDAY 19
٣ جمادى الاول

TUESDAY 20
٥ جمادى الاول

Step 0: we marked the cells with X⁰ which can not be merged as per rules of equivalence of states. Since the cells are marked in step 0, so we need another iteration to mark cells further.

Step 1:

$$\delta[(A, D), 0] = (B, D) = ?$$

$$\delta[(A, D), 1] = (C, D) = ?$$

$$\delta[(A, C), 0] = (B, E) = X^1$$

$$\delta[(A, C), 1] = (C, D) = ?$$

$$\delta[(A, B), 0] = (B, E) = X^1$$

$$\delta[(A, B), 1] = (C, D) = ?$$

APRIL					
M	T	W	T	F	S
5	6	7	1	2	3
12	13	14	8	9	10
19	20	21	15	16	17
26	27	28	22	23	24
			29	30	

$$\delta[(B,D),0] = (E,D) = X^1$$

$$\delta[(B,D),1] = (D,D) = ?$$

$$\delta[(B,C),0] = (E,F) = ?$$

$$\delta[(B,C),1] = (D,D) = ?$$

$$\delta[(C,D),0] = (F,D) = X^1$$

$$\delta[(C,D),1] = (D,D) = ?$$

$$\delta[(E,F),0] = (E,F) = ?$$

$$\delta[(E,F),1] = (E,F) = ?$$

Since cells are again marked in step 1, so we need another iteration

21 WEDNESDAY
٢ جمادى الاول

22 THURSDAY
٤ جمادى الاول

Step 2: $\delta[(A,D),0] = (B,D) = X^2$

$$\delta[(A,D),1] = (C,D) = X^2$$

$$\delta[(B,C),0] = (E,F) = ?$$

$$\delta[(B,C),1] = (D,D) = ?$$

$$\delta[(E,F),0] = (E,F) = ?$$

$$\delta[(E,F),1] = (E,F) = ?$$

Since cells are marked again, so we need another iteration

Step 3: $\delta[(B,C),0] = (E,F) = ?$

APRIL					
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		1	2	3	4
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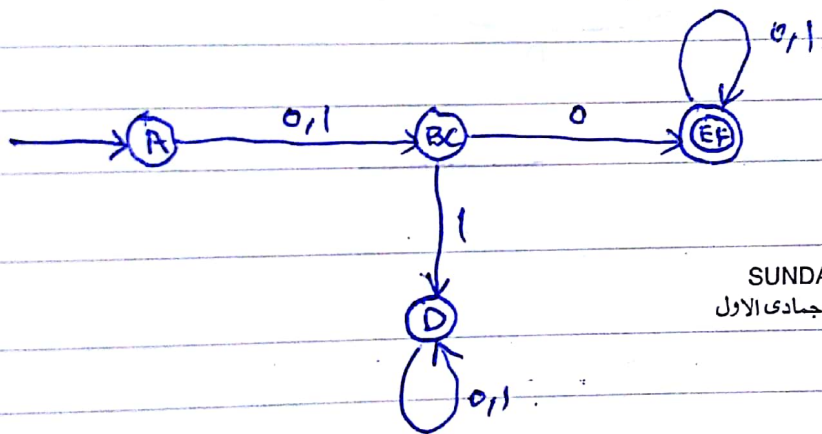
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$$\delta[(B,c),1] = (D,D) = ?$$

$$\delta[(E,F),0] = (E,F) = ?$$

$$\delta[(E,F),1] = (E,F) = ?$$

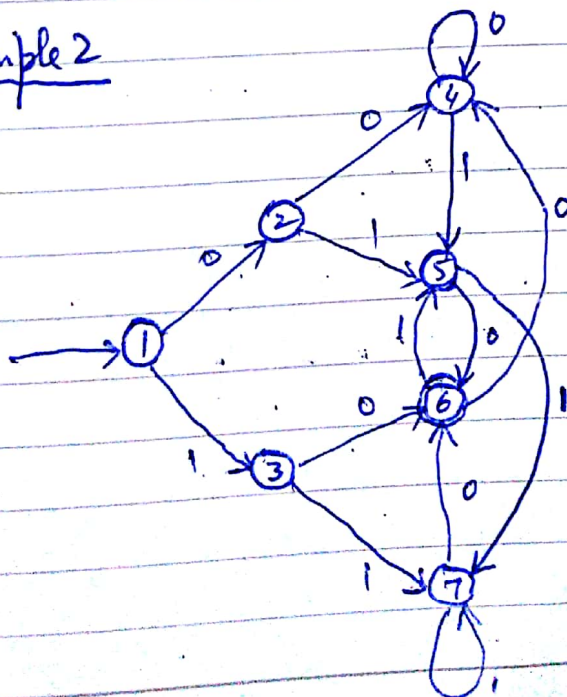
iteration stops here, since no any cell is marked in step 3 and (B,c) and (E,F) are remained as unmarked cells, so we can merge them and final minimized DFA is as below.



FRIDAY 23
جمادی الاول ۸

SUNDAY 25 جمادی الاول ۱۰
SATURDAY 24 جمادی الاول ۹

Example 2



from Book

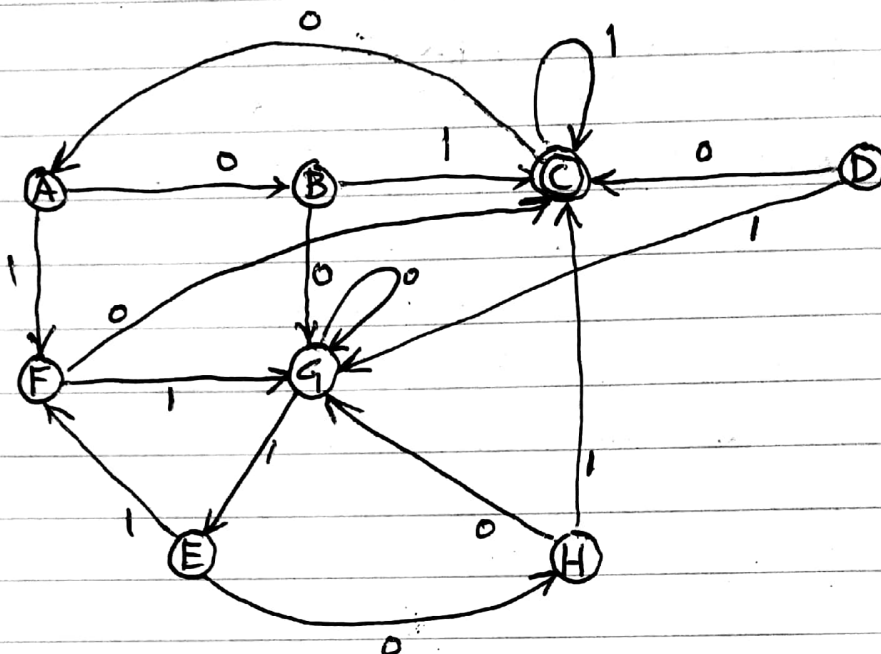
Solve it in class

جمادى الاول ١٣٣١

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APRIL						
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19	20	21	22	23	24	25
26	27	28	29	30		

H.W



Evening

26 MONDAY
١١ جمادى الاول

27 TUESDAY
١٢ جمادى الاول

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Evening

APRIL						
M	T	W	T	F	S	S
			1	2	3	4
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Statement of Pumping Lemma

If language L is regular, then $\forall z \in L$ where $|z| \geq n$ (n is number of states in FSM of L).

\exists strings u, v, w such that $z = uvw$ where

$$|uv| \leq n$$

$$|v| > 0$$

for any $i \geq 0, uv^i w \in L$

Pumping lemma extraction

Says

→ if an infinite language is regular, it is defined by DFA.

→ The DFA (then must) have some finite number of states say n .

→ Since language is infinite, some strings of language must have length $\geq n$.

→ For a string of length $\geq n$ accepted by the DFA, we would have to walk through DFA that must have a cycle.

→ Repeating the cycle on arbitrary number of times must yield another string accepted by the DFA \Rightarrow Pumping property.

Negation of Pumping Property (Contrapositive/Contradiction)

$\exists z \in L ; |z| \geq n$

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MAY						
M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

$\exists u, v, w$ where $z = uvw$; Pumping property
 $|uv| \leq n$ and $|v| > 0$
 $\exists i \geq 0$; such that $uv^i w \notin L$
 then L is not regular

So we conclude
 Regular languages \rightarrow Pumping property
 \neg Pumping property $\rightarrow \neg$ Regular languages

General Results :-

- 1- Pumping lemma used to prove that given infinite language is not regular.
- 2- Pumping property is for infinite languages.
- 3- Pumping lemma can not be used to prove that a given language is regular.

✓ Example

$P =$ palindrome (non regular language)

1. Let P be a regular language
2. P will have FSM of n state
3. $z = 0^n 1 0^n$ and $z \in L$ & $|z| \geq n$

Then

4. $|z| = |0^n 1 0^n| \geq n$

$n+1+n \geq n$

$2n+1 \geq n$

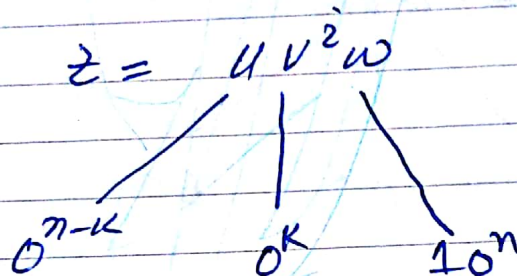
proved

MAY						
M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
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non



$\therefore k > 0$

Then

$$\begin{aligned} 5 - uv^i w &= u^{n-k} v^{2k} w^n \\ &= u^{n+k(i-1)} w^n \end{aligned}$$

Then $n+k(i-1) \neq n$ for $i \geq 2$

Hence P is a non regular language

MONDAY 3
١٨ جمادى الاول
TUESDAY 4
١٩ جمادى الاول

Example

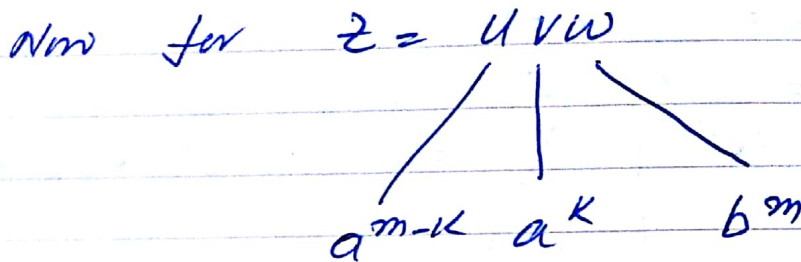
$$L = \{a^n b^n\}$$

1. Let L be a R.L
2. having FSM $\neq m$ states
3. $z = a^m b^m$
then since $z \in L$, so
4. $|z| \geq m$

$$|a^m b^m| \geq m \Rightarrow m+m \geq m \Rightarrow 2m \geq m$$

proved

MAY						
M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30



5. $UV^iW \in L$, so

$$a^{m-k} a^{ik} b^m \Rightarrow a^{m+k(i-1)} b^m$$

which gives for $i \geq 2$, $m+k(i-1) \neq m$

So L is not regular.

5 WEDNESDAY
٢٠ جمادى الاول

Example

$$L = \{xx \mid x \in \{0,1\}^*\}$$

6 THURSDAY
٢١ جمادى الاول

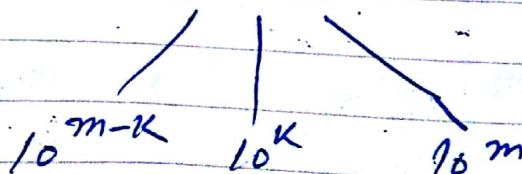
- (1) - det of bc R.R
- (2) - FSM of m states
- (3) - $z = (10)^m (10)^m$
- (4) - $z \in L$, then $|z| \geq m$

$$|(10)^m (10)^m| \geq m$$

$$2m + 2m \geq m$$

$$4m \geq m$$

(5) - $z = UVW$



MAY						
M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

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(6) - $uv^2w \in L$, then

$$10^{m-k} 10^{2k} 10^m \Rightarrow 10^{m+k(2-1)} 10^m$$

for $i \geq 2$ $m+k(i-1) \neq m$

Hence L is not regular.

Example

$$L = 1^n^2 \text{ where } n \geq 0$$

Evening

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MAY 2010

MAY						
M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Example (Repeat) ~~the same as the previous~~

$$z = 10^a 10^b 10^c$$

$$z = UVW$$

$$10^a \quad 0^b \quad 0^{m-a-b} 10^m$$

$$UV^2W \in L$$

$$10^a 0^{2b} 0^{m-a-b} 10^m$$

$$\Rightarrow 10^{a+2b+m-a-b} 10^m \Rightarrow 10^{m+b(2-1)} 10^m$$

So for $i \geq 2$, $m + b(i-1) \neq m$

Hence non regular language.

10 MONDAY
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11 TUESDAY
جمادی الاول ۲۶

H.W

$$L = \{ a^n \text{ where } n \text{ is prime no.} \}$$

Evening

Example $L = 1^{n^2}$ where $n \geq 0$

1 - det L is R.L

2 - FSM \neq N states

3 - $Z = 1^{N^2} \Rightarrow \{1, 1^4, 1^9, 1^{16}, \dots\}$

$1^{N^2} \geq m$ proved

5 -

~~condition~~
 $Z_{-1} = 1^{(N-1)^2}$

$Z = 1^{N^2}$

$Z_{+1} = 1^{(N+1)^2}$

Now $(Z) |UVW| = n^2 < |UV^2W| = |UVUVW|$

$< n^2 + |V| \because 0 < |V| \leq N$

$< n^2 + n$

$< n^2 + n + n + 1 \because$ square Z_{++}

$< n^2 + 2n + 1 \because$ square Z_{++}

$< (n+1)^2$

$< Z_{+1}$

As UV^2W does not belong to L , So L is not Regular