			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Equivalence of States :-

Given $P, Q \in Q$, then

→ if $P = f$ & $Q \neq f$ ⇒ P & Q are distinguishable and we can not merge them.

→ if (P, Q) are non-distinguishable, then they can be merged, means both should be final states or non-final states

Example 1 (DFA Minimization)

DFA → [Algo of ES] → Minimized DFA

or

optimal minized DFA

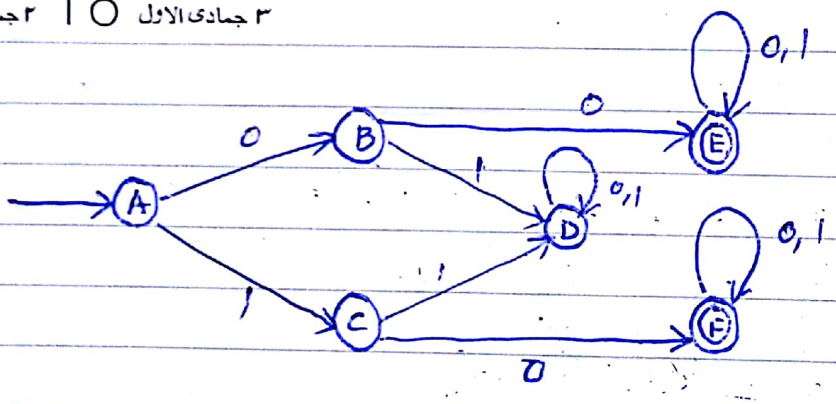
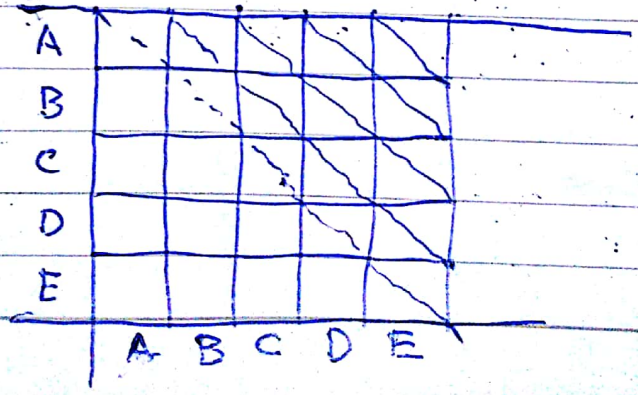


Fig A

Then we build a N×N Step Table



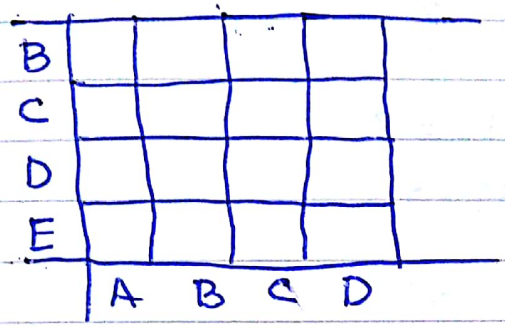
Since

(B,D) ≡ (D,B)

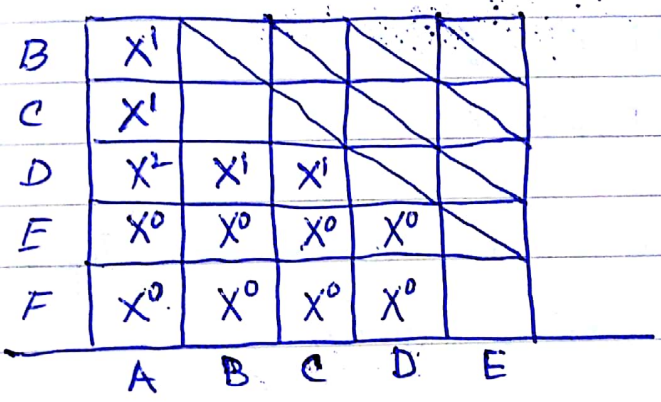
(AA) ≡ A

So, we don't need the crossed portion of the table

Hence the rest of the step table becomes



By applying the Concept, the step table of fig # becomes



Step 0: we marked the cells with X^0 which can not be merged as per rules of equivalence of states. Since the cells are marked in step 0, so we need another iteration to marked cells further.

Step 1:

$$\delta[(A,D),0] = (B,D) = ?$$

$$\delta[(A,D),1] = (C,D) = ?$$

$$\delta[(A,C),0] = (B,F) = X_1$$

$$\delta[(A,C),1] = (C,D) = ?$$

$$\delta[(A,B),0] = (B,E) = X^1$$

$$\delta[(A,B),1] = (C,D) = ?$$

$\delta[(B,D),0] = (E,D) = X^1$

$\delta[(B,D),1] = (D,D) = ?$

$\delta[(B,C),0] = (E,F) = ?$

$\delta[(B,C),1] = (D,D) = ?$

$\delta[(C,D),0] = (F,D) = X^1$

$\delta[(C,D),1] = (D,D) = ?$

$\delta[(E,F),0] = (E,F) = ?$

$\delta[(E,F),1] = (E,F) = ?$

Since cells are again marked in step 1, so we need another iteration

Step IV: $\delta[(A,D),0] = (B,D) = X^2$

$\delta[(A,D),1] = (C,D) = X^2$

$\delta[(B,C),0] = (E,F) = ?$

$\delta[(B,C),1] = (D,D) = ?$

$\delta[(E,F),0] = (E,F) = ?$

$\delta[(E,F),1] = (E,F) = ?$

Since cells are marked again, so we need another iteration

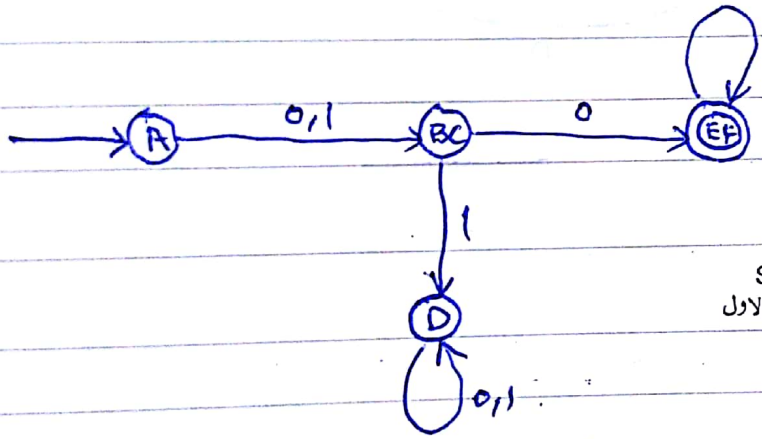
Step 3: $\delta[(B,C),0] = (E,F) = ?$

$\delta[(B,C),1] = (D,D) = ?$

$\delta\{(E,F),0\} = (E,F) = ?$

$\delta\{(E,F),1\} = (E,F) = ?$

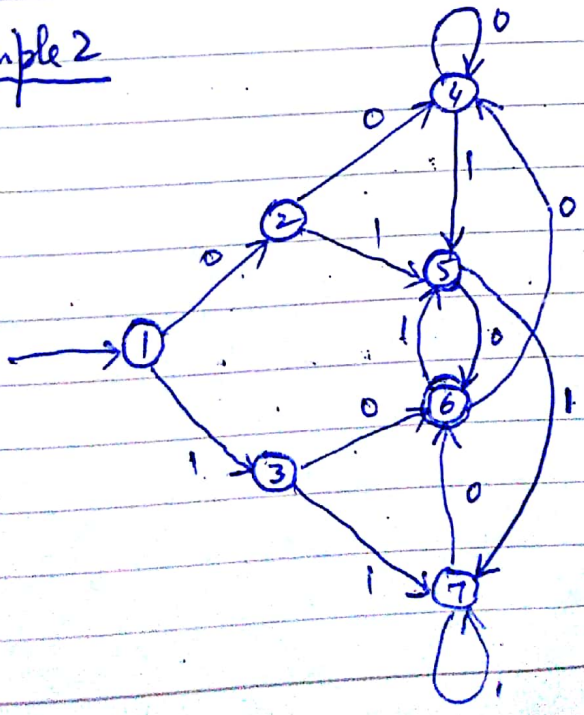
Iteration stops here, since no any cell is marked in step 3 and (B,C) and (E,F) are remained as unmarked cells, so we can merge them and final minimized DFA is as below.



Example 2

from Book

Solve it in class



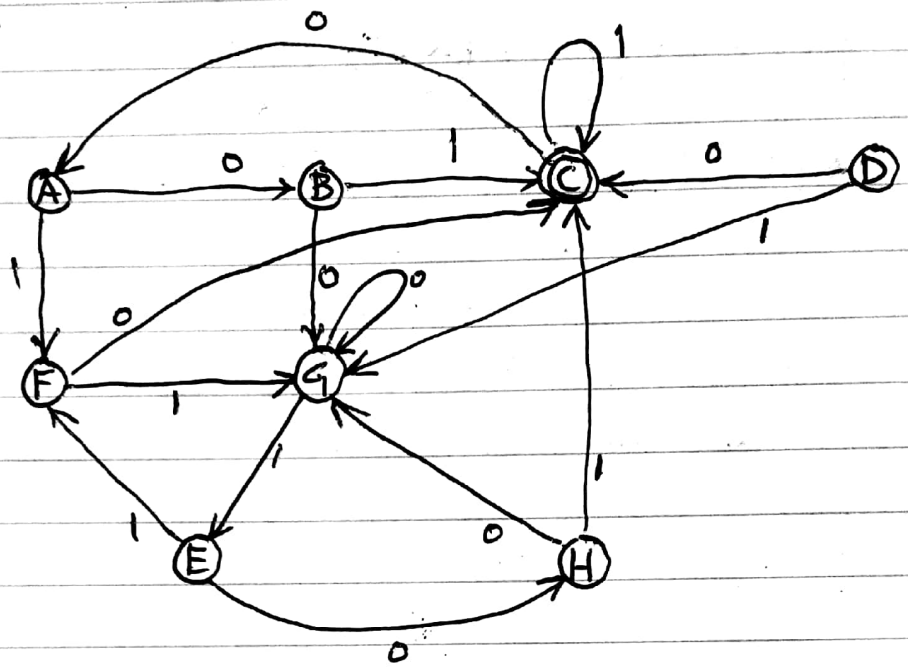
جمادى الاول ١٤٣١

APRIL 2010

APRIL

M	T	W	T	F	S	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

H.W



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Evening

26 MONDAY
١١ جمادى الاول

27 TUESDAY
١٢ جمادى الاول

08
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05
Evening

Statement of Pumping Lemma

If language L is regular, then $\forall z \in L$ where $|z| \geq n$ (n is number of states in FSM of L).
$\exists$ strings u, v, w such that $z = uvw$ where

$$|uv| \leq n$$
$$|v| > 0$$
$$\text{for any } i \geq 0,\ uv^i w \in L$$

Pumping Lemma extraction

Says

→ if an infinite language is regular, it is defined by DFA.

→ The DFA (then must) have some finite number of states Say n.

→ Since language is infinite, some strings of language must have length $\geq n$

→ for a string of length $\geq n$ accepted by the DFA, we would have to walk through DFA that must have a cycle.

→ Repeating the cycle on arbitrary number of times must yield another string accepted by the DFA ⇒ Pumping property.

WEDNESDAY 28 ۱۳ جمادی الاول

THURSDAY 29 ۱۴ جمادی الاول

Negation of Pumping Property (Contrapositive/ Contradiction)

$\exists z \in L$; $|z| \geq n$

$\exists$ u, v, w where $z = uvw$; $|uv| \leq n$ and $|v| > 0$

$\exists i \geq 0$; such that $\boxed{uv^2w \notin L}$ ← Pumping property

then L is not regular

So we Conclude

Regular languages ⟶ Pumping property

¬ Pumping property ⟶ ¬ Regular languages

Evening

General Results :-

1- Pumping lemma used to prove that given infinite language is not regular.

30 FRIDAY ١٥ جمادى الاول

1 SATURDAY ١٦ جمادى الاول 2 SUNDAY ١٧ جمادى الاول

2- Pumping property is for infinite langues

3- Pumping lemma can not be used to prove that a given languages is regular

06 09 10 11 12 01 02 03 04 05

Example

P = palindrome (non Regular langes)

1. Let P be a Regular Language
2. P will have FSM of n state
3. $z = 0^n 1 0^n$ and $z \in L$ & $|z| \geq n$
4. Then

$|z| = |0^n 1 0^n| \geq n$

$n + 1 + n \geq n$

$2n + 1 \geq n$ proved

Evening

now

$z = uv^i w$ $\quad \therefore k > 0$

0^{n-k} $\quad 0^k$ $\quad 10^n$

Then

5- $uv^i w = 0^{n-k}\, 0^{ik}\, 10^n$

$= 0^{n+k(i-1)}\, 10^n$

Then $n + k(i-1) \neq n$ for $i \geq 2$

Hence P is a non regular language

MONDAY 3 جمادى الاول ١٨

TUESDAY 4 جمادى الاول ١٩

Example $L = \{a^n b^n\}$

1- let L be a R.L

2- having FSM of m states

3- $z = a^m b^m$

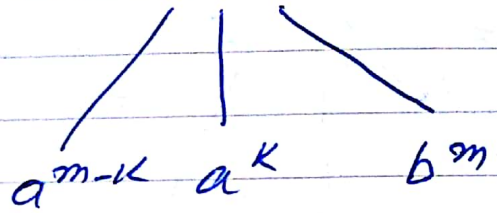
then since $z \in L$, so

4- $|z| \geq m$

$|a^m b^m| \geq m \Rightarrow m + m \geq m \Rightarrow 2m \geq m$

proved

Now for $z = uvw$



5- $uv^iw \in L$, so

$$a^{m-k} a^{ik} b^m \Rightarrow a^{m+k(i-1)} b^m$$

which gives for $i \geq 2$, $m + k(i-1) \neq m$

So L is not regular.

5 WEDNESDAY ۲۰ جمادی الاول
6 THURSDAY ۲۱ جمادی الاول

Evening

Example $L = \{ x\,x \mid x \in \{0,1\}^* \}$

(1)- Let L be R.R

(2)- FSM of m states

(3)- $z = (10)^m (10)^m$

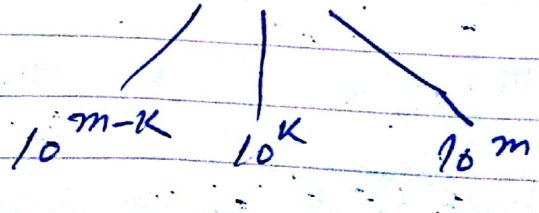
(4)- $z \in L$, then $|z| \geq m$

$$|(10)^m (10)^m| \geq m$$

$$2m + 2m \geq m$$

$$4m \geq m$$

(5)- $z = uvw$



Evening

(6)- $uv^2w \in L$, then

$10^{m-k}\ 10^{2k}\ 10^m \Rightarrow 10^{m+k(2-1)}\ 10^m$

for $i \geq 2$ $m + k(i-1) \neq m$

Hence L is not Regular.

Example $L = 1^{m^2}$ where $m \geq 0$

1- Let L is R.L

2- FSM $\$$ on states

3- $z = 1^{m^2}$

4- $z \in L$, then $|z| \geq m$

$|1^{m^2}| \geq m$

$m^2 \geq m$ proved

5- $z = uvw$

$1^a \quad 1^b \quad 1^{m^2-a-b}$

6- $uv^2w \in L$, then

$1^a\ 1^{ib}\ 1^{m^2-a-b} \Rightarrow 1^{a+ib+m^2-a-b}$

$\Rightarrow 1^{m^2+b(i-1)}$, Hence for $i \geq 1$

$m^2 + b(i-1) \neq m$, so L is non Regular

جمادی الاول ۱۴۳۱

MAY 2010

MAY

M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Example (Repeat)

$z = 1 0^m / 0^m$

$z = uvw$

$1 0^a \quad 0^b \quad 0^{m-a-b} 1 0^m$

uv^2wel

$1 0^a 0^{ib} 0^{m-a-b} 1 0^m$

$\Rightarrow 1 0^{a+ib+m-a-b} 1 0^m \Rightarrow 1 0^{m+b(i-1)} 1 0^m$

so for $i \geq 2$, $m + b(i-1) \neq m$

Hence non Regular language.

10 MONDAY ۲۵ جمادی الاول

11 TUESDAY ۲۶ جمادی الاول

H.W $L = \{ a^n$ where n is prime no $\}$