

JANUARY						
M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

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→ Definition of finite Automaton

A finite automaton, or finite state machine (FA) is a five tuple $(Q, \Sigma, q_0, A, \delta)$ where

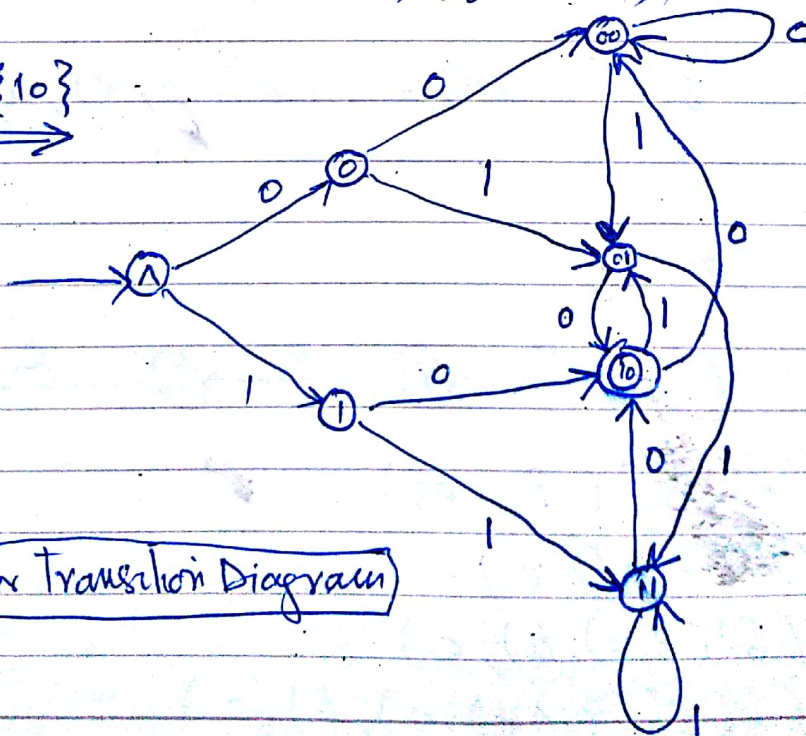
- Q is finite set of states
- Σ is finite alphabet of input symbols
- $q_0 \in Q$ (the initial state)
- $A \subseteq Q$ (the set of accepting states)
- δ is a function from $Q \times \Sigma$ to Q (the transition function)

for any element q of Q and any symbol $a \in \Sigma$, we interpret $\delta(q, a)$ as the state to which the FA moves, if it is in state q and receives the input a .

Then Mathematically we can say that

$M = (Q, \Sigma, q_0, A, \delta)$ be an FA

✓ for $\{0,1\}^* \{10\}$



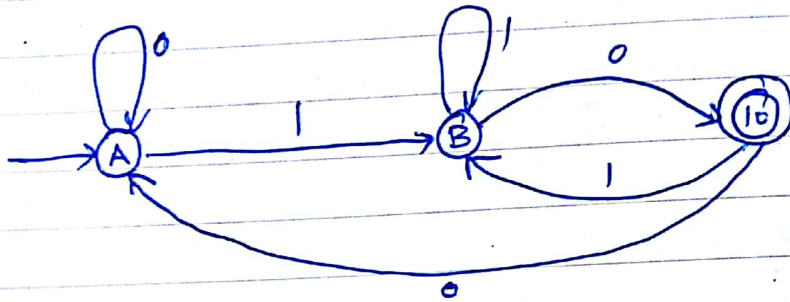
FA or Transition Diagram

	input		
	0	1	
q0	q1	q1	
q1	q2	q2	
q2	q3	q3	
q3	q0	q0	
q3	q3	q3	
q3	q3	q3	
q3	q3	q3	
q3	q3	q3	

Transition Table

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- Simplified or reduced form



Recursive defn

→ Defn: The extended transition function δ^*

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an FA, we define the function

$$\delta^*: Q \times \Sigma^* \rightarrow Q$$

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as follows

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- for any $q \in Q$, $\delta^*(q, \epsilon) = q$
- for any $q \in Q$, $y \in \Sigma^*$, and $a \in \Sigma$

$$\delta^*(q, ya) = \delta(\delta^*(q, y), a)$$

Example

let

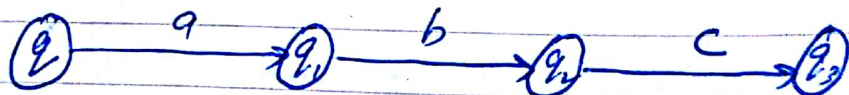


fig 3.4

$$\delta^*(q_0, abc)$$

$$= \delta(\delta^*(q_0, ab), c)$$

$$= \delta(\delta(\delta^*(q_0, a), b), c)$$

$$= \delta(\delta(\delta(\delta^*(q_0, \epsilon), a), b), c)$$

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$$= \delta(\delta(\delta(q, a), b), c)$$

$$= \delta(\delta(q, ab), c)$$

$$= \delta(q_2, c)$$

$$= q_3 \quad \checkmark$$

Now extended concept for x and y string which contains alphabets

$$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$$

→ Definition 3.4: Acceptance by an FA

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an FA. WEDNESDAY 20
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A string $x \in \Sigma^*$ is accepted by M if THURSDAY 21
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$\delta^*(q_0, x) \in A$. If a string is not accepted,
 state q

we say it is rejected by M . The language accepted by M , or the language recognized by M , is the set

$$L(M) = \{x \in \Sigma^* \mid x \text{ is accepted by } M\}$$

If L is any language over Σ , L is accepted or recognized by M if and only if $L = L(M)$

"
* The Power of machine does not lie in the number of strings it accepts" but in accepting some and rejecting others.

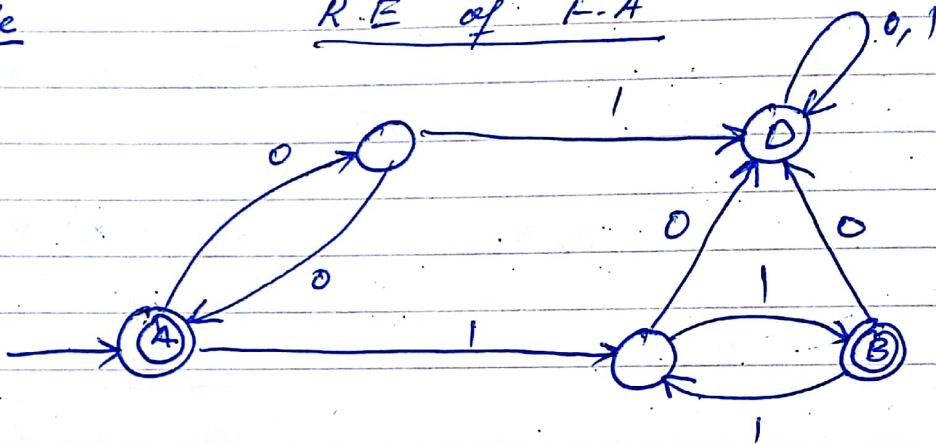
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Theorem

A language L over the alphabet Σ is regular iff there is an FA with input alphabet Σ that accepts L .

Example

R.E of F.A



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R.E = $(00)^*(11)^*$

Example

A finite Automaton M accepting $\{a, b\}^* \{baaa\}$ can be seen in book

Example

R.E to F.A

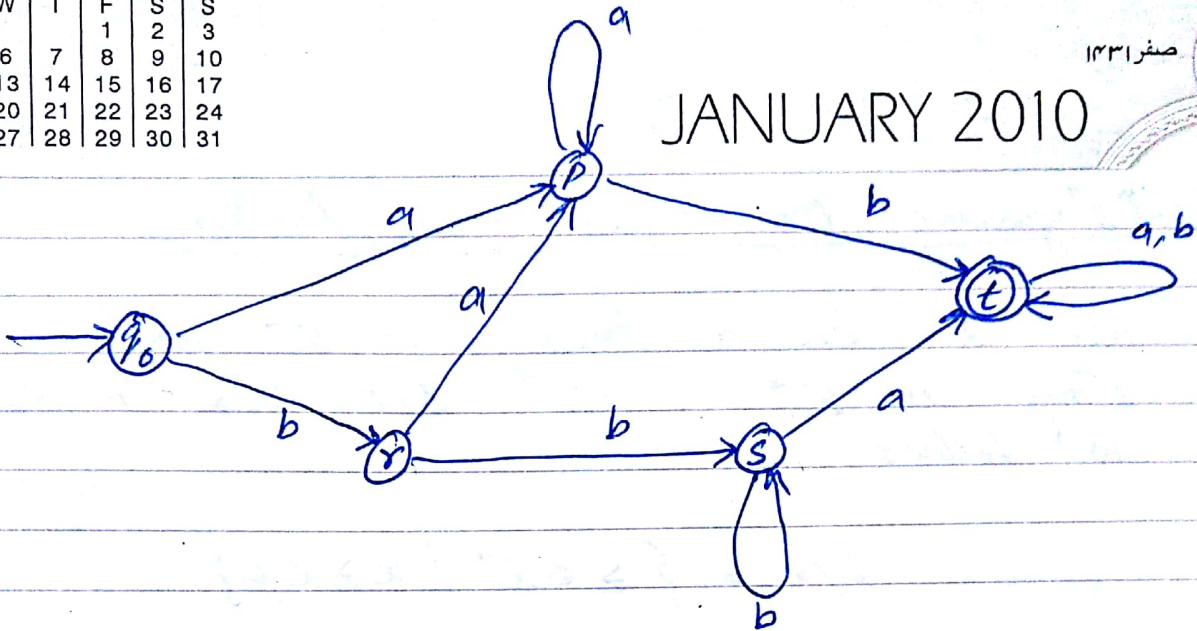
Language L of All strings in $(a, b)^*$ That contain at least one of two substring ab and bb or $(a+b)^*(ab+bb)(a+b)^*$

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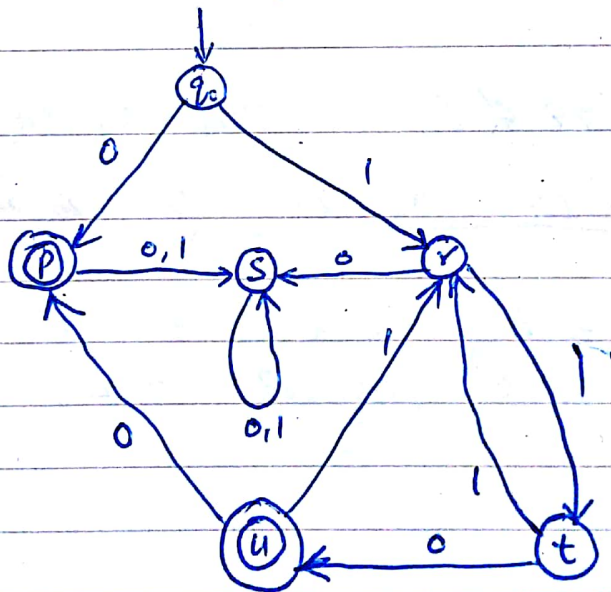


Example

$$\delta = (11 + 110)^* 0$$

$$\Sigma = \{0, 1\}$$

i) - $0 \in L$ & $1 \notin L$, $10 \notin L$, $110 \in L$, $1110 \notin L$



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Distinguishing One String from Another

Let L be a language in Σ^* and x any string in Σ^* . The set L/x is defined as follows

$$L/x = \{z \in \Sigma^* \mid xz \in L\}$$

Two strings x and y are said to be distinguishable with respect to L if $L/x \neq L/y$. Any string z that is in one of two sets but not the other (i.e. for which $xz \in L$

7 WEDNESDAY ۱۱ صفر and $yz \notin L$, or vice versa) is said to

8 THURSDAY ۱۲ صفر distinguish x and y with respect to L .

If $L/x = L/y$, x and y are indistinguishable with respect to L

✓ Example

the language L of all strings in $(0,1)^*$ that end in 10 .

strings $\frac{x}{00}$ and $\frac{y}{01}$ are distinguishable with respect to L because if $z = 0$, then $\frac{000}{x z} \notin L$ and $\frac{010}{y z} \in L$

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which gives

$$xz \notin L \neq yz \in L$$

$$L/x \neq L/y$$

Similarly

if two strings are $\frac{0}{x}$ and $\frac{00}{y}$

then L/x and L/y are equal and indistinguishable (Now in this case the property of ending in '0' depends on 'z' string)

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Lemma

Suppose $L \subseteq \Sigma^*$ and $M = (Q, \Sigma, q_0, A, \delta)$ is an FA recognizing L . If x and y are two strings in Σ^* that are distinguishable with respect to L , then

$$\delta^*(q_0, x) \neq \delta^*(q_0, y)$$

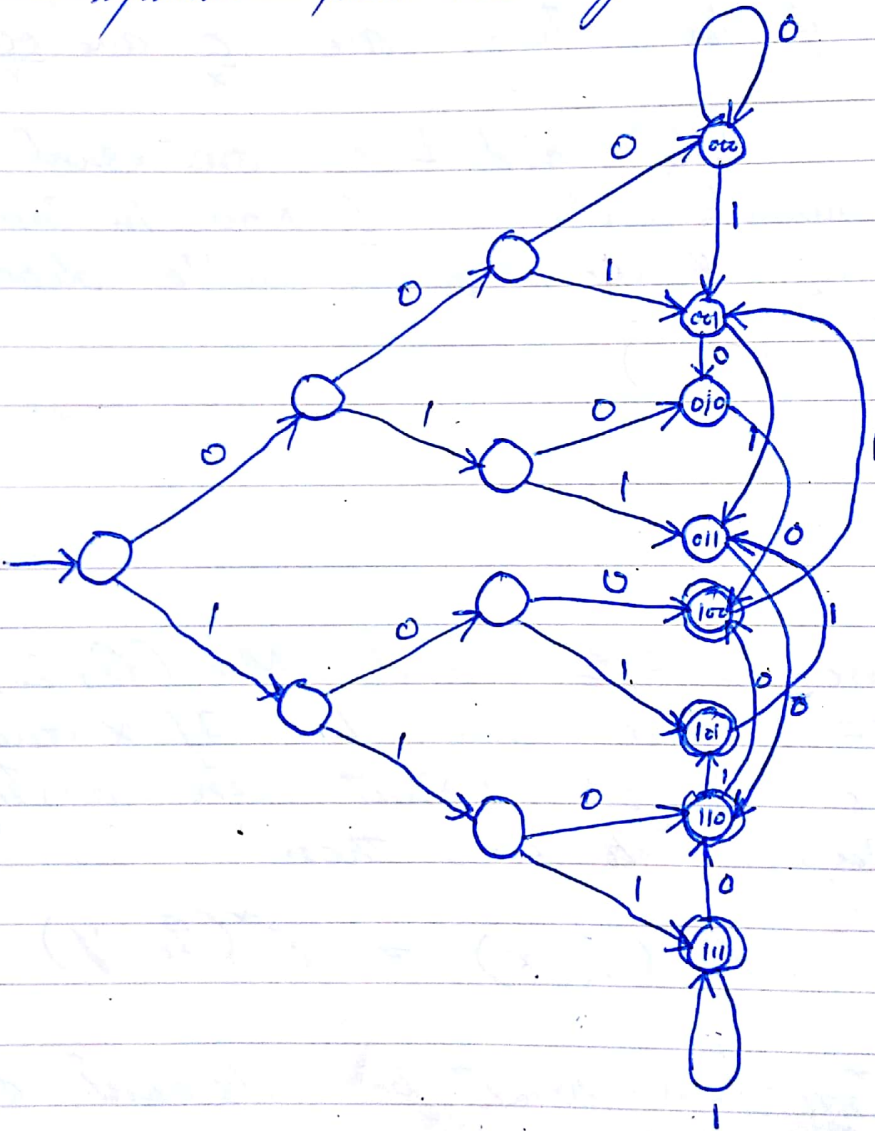
(their transitions must be different on FA)

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15	16	17	18	19	20	21
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✓ FA Example of language L_n

Suppose $n \geq 1$ and let

$$L_n = \{x \in (0,1)^* \mid |x| \geq n \text{ and the } n\text{th symbol from the right in } x \text{ is } 1\}$$



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Evening Theorem the language pal of palindromes over the alphabet $\{0,1\}$ can not be accepted by any FA and is therefore not regular