Context Free Grammers

→ Every CFL has CFG

→ A Context free grammer is simple recursive method of specifying grammar rules by which strings in a language can be generated.

→ To demonstrate that a CFG generates a language we must show two things

① - Every string in the language can be derived from grammar.

② - No other string can be generated from grammar.

Gen. Example of CFG

Set of rules / Productions

<Sentence> → <NP> <VP>
<NP> → <Article> <Noun>
<VP> → <Verb> <Adverb>
<Article> → a
<Article> → an
<Noun> → dog
<Noun> → man
<Verb> → is talking
<Verb> → is barking
<Adverb> → politely
<Adverb> → loudly

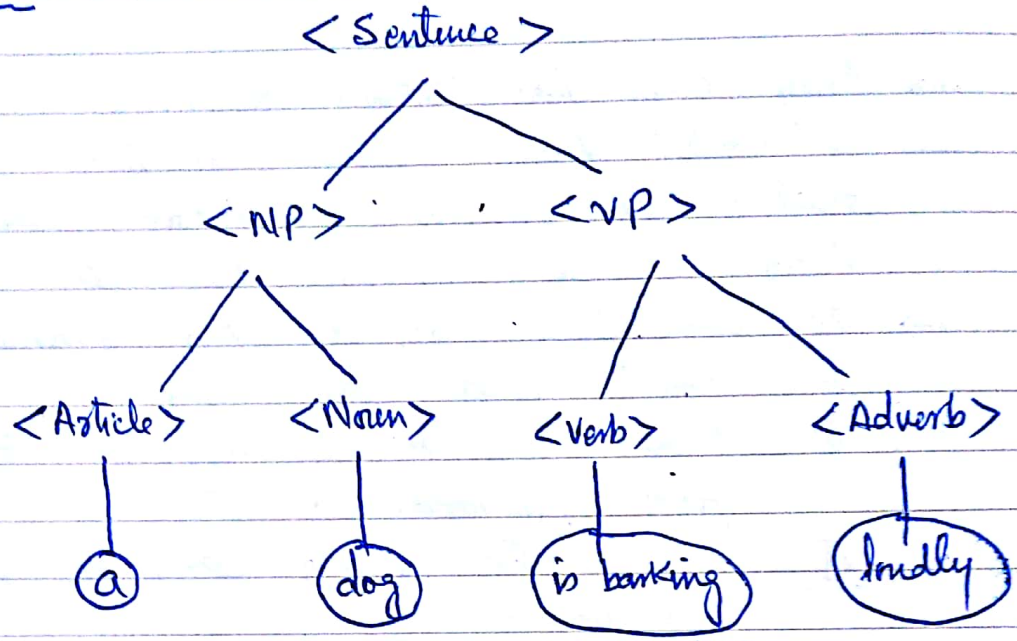
Now sentence is "a dog is barking loudly"

MAY

M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

08 09 10 11 12 01 02 03 04 05 Evening

Parse Tree



∴ Terminals are encircled

14 FRIDAY ۲۹ جمادی الاول

15 SATURDAY ۳۰ جمادی الاول 16 SUNDAY ۱ جمادی الثانی

08 09 10 11 12 01 02 03 04 05 Evening

Defn of CFG

A Context free grammar is a 4-tuple $G=\{V,\Sigma,S,P\}$ where V and Σ are disjoint finite sets. S is an element of V, and P is a finite set rules/Productions of the form $A \to \alpha$, where $A \in V$ and $\alpha \in (V \cup \Sigma)^*$

Defn of CFL

Let $G=\{V,\Sigma,S,P\}$ be a CFG. The language generated by G is $L(G)=\{x \in \Sigma^* \mid S \overset{*}{\Rightarrow}_G x\}$

A language L is CFL if there is a CFG 'G'

So that $L = L(G)$

Example (The language of Algebraic Expressions)

$$S \to S+S \mid S-S \mid S*S \mid S/S \mid (S) \mid a$$

the string $a+(a*a)/a-a$ Can be obtained from the derivation as

$$S \Rightarrow S-S \Rightarrow S+S-S \Rightarrow a+S-S \Rightarrow a+S/S-S$$
$$\Rightarrow a+(S)/S-S \Rightarrow a+(S*S)/S-S$$
$$\Rightarrow a+(a*S)/S-S \Rightarrow a+(a*a)/S-S$$
$$\Rightarrow a+(a*a)/a-S \Rightarrow a+(a*a)/a-a$$

MONDAY ٢ جمادى الثانى 17

TUESDAY ٣ جمادى الثانى 18

Example $R.E = a^*$

$$S \to aS \mid \Lambda$$

Example $R.E = a^* b^*$

$$S \to AB$$
$$A \to aA \mid \Lambda$$
$$B \to bB \mid \Lambda$$

Example $(a+b)^*$

$$S \to aS \mid bS \mid \Lambda$$

MAY

M	T	W	T	F	S	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

08
09
10
11
12
01
02
03
04
05
Evening

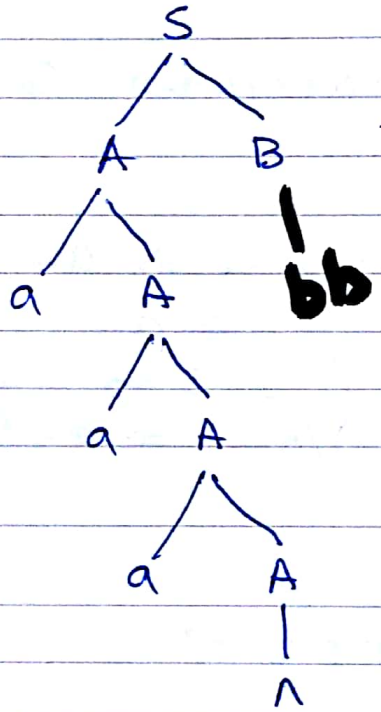
Example a^*bb

$S \rightarrow AB$

$A \rightarrow aA \mid \Lambda$

$B \rightarrow bb$

for aaabb



19 WEDNESDAY ۴ جمادی الثانی

20 THURSDAY ۵ جمادی الثانی

08
09
10
11
12
01
02
03
04
05
Evening

Example (String aa must be included)

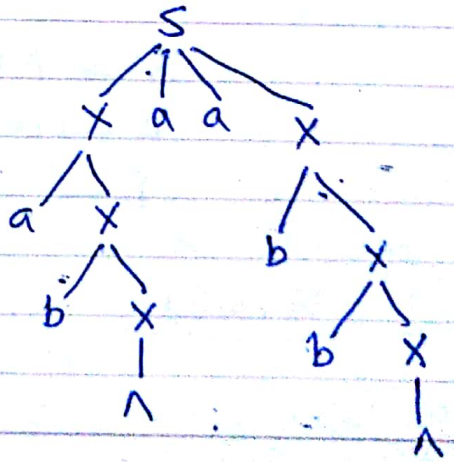
$S \rightarrow XaaX$

$X \rightarrow aX$

$X \rightarrow bX$

$X \rightarrow \Lambda$

for abaabb



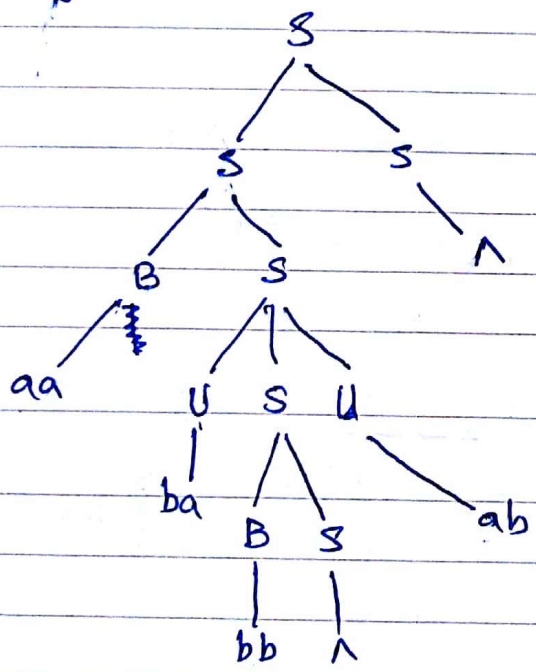
Example (even no. of a's and even no. of b's)

$S \rightarrow SS \mid BS \mid SB \mid \Lambda \mid USU$

$B \rightarrow aa \mid bb$

$U \rightarrow ab \mid ba$

for string aababbab



(H.W) Example $S \rightarrow (S) \mid S \supset S \mid \sim S \mid P \mid Q$

Derive 13 letter word $(\sim\sim P \supset (P \supset \sim\sim Q))$

Example $L = \{ x \in \{0,1\}^* \mid n_0(x) = n_1(x) \}$

$S \rightarrow SS \mid 0S1 \mid 1S0 \mid \Lambda$

for string 1000110

08 $S \Rightarrow SS \Rightarrow SSS \Rightarrow 1S0\ 0S1\ 0S1$

09 $\Rightarrow 1\Lambda0\ 00S11\ 0S1$

10 $\Rightarrow 100\Lambda11\ 0\Lambda1 \Rightarrow 1000110\,1$

11

12 **Theorem**

01 if L_1 and L_2 are CFL's, then the

02 languages $L_1 \cup L_2$, $L_1 L_2$ and L_1^* are also CFLs.

03

04

05 **Example** (A CFG equivalent to R.E)

Evening $(011+1)^*(01)^*$

24 MONDAY ٩ جمادى الثانى

25 TUESDAY ١٠ جمادى الثانى

$A \rightarrow 011 \mid 1$ } (generates $\{011, 1\}$).

$B \rightarrow AB \mid \Lambda$ } generates $(011, 1)^*$

$A \rightarrow 011 \mid 1$

$C \rightarrow DC \mid \Lambda$ } generates $(01)^*$

$D \rightarrow 01$

finally

$S \rightarrow BC$

$B \rightarrow AB \mid \Lambda$

$A \rightarrow 011 \mid 1$

$C \rightarrow DC \mid \Lambda$

$D \rightarrow 01$

Similarly CFG's can be obtained simply

Through some FA by applying simple logic which can be seen in book. So every R.L is CFL.

Derivation & Ambiguity

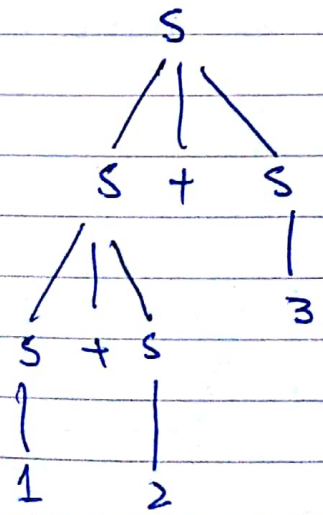
Derivation

LMD — { left most NT expanded first }

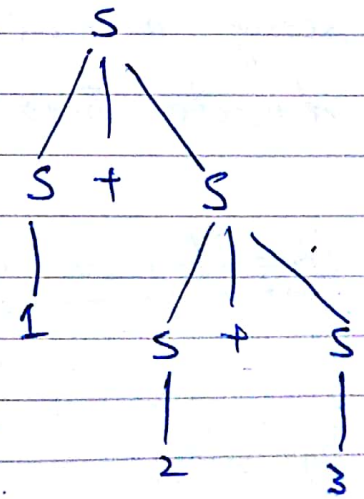
RMD — { Right most NT expanded first }

$S \rightarrow S+S \mid S*S$

$S \rightarrow 0|1|2|3|\cdots|9$



LMDT

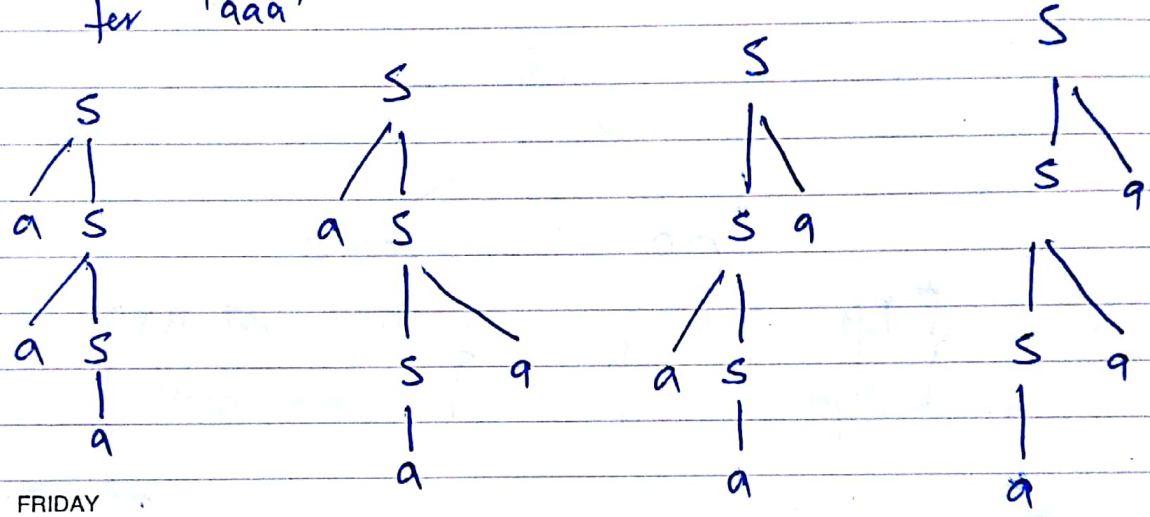


RMDT

Example (Ambiguity)

$S \rightarrow aS \mid Sa \mid a$

for 'aaa'



28 FRIDAY ۱۳ جمادی الثانی

29 SATURDAY ۱۴ جمادی الثانی

30 SUNDAY ۱۵ جمادی الثانی

"A CFG is ambiguous if there is at least one string in $L(G)$ having two or more distinct derivation trees"

Unambiguous CFG

$S \rightarrow S+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (S) \mid a$

$S \rightarrow S+S \mid S*S \mid (S) \mid a$