Homework # 1
Theory of Automata and Formal Languages

Total Marks: 130 Course Code: CS-3131

Note: All questions carry equal marks.

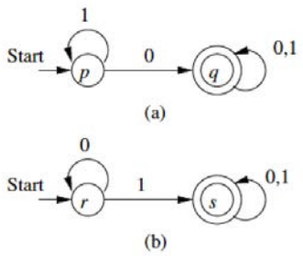
Q1: Prove the following Theorem: If $x \geq 4$ then $2^x \geq x^2$.
Q2: Prove the following Theorem: Let x be a real number. Then $\lfloor x \rfloor = \lceil x \rceil$ if and only if x is an integer.
Q3: Discuss about proof by contradiction and proof by contrapositive.
Q4: Prove the following by induction method.
For all $n \geq 0$:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Q5: Write down the definition of DFA.
Q6: Design a DFA to accept the language L= {w | w has both an even number of 0's and an even number of 1's}.
Q7: Give DFA accepting the following language over the alphabet {0,1}. The language is the set of all strings beginning with a 1 that when interpreted as a binary integer is a multiple of 5. For example strings 101, 1010 and 1111 are in the language while 0, 100 and 111 are not.
Q8: Provide the definition of an NFA.
Q9: Write regular expressions for the following languages:
The set of strings over alphabet {a,b,c} containing at least one a and at least one b .
The set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.
The set of all strings of 0's and 1's not containing 101 as a substring.
Q10: Give English descriptions of the languages of the following regular expressions:

a) $(1 + \varepsilon)(00^*1)0^*$
b) $(0^*1^*)^*000(0 + 1)^*$
c) $(0 + 10)^*1^*$

Q11: Define and explain the Extended Transition Function of DFA and NFA with respective examples.
Q12: Design a DFA for language Ln by supposing n =3, where overall $n \geq 1$ and $Ln = \{x \in (0,1)^* \mid |x| \geq n \text{ and } nth\ symbol\ from\ the\ right\ in\ x\ is\ 1\}$
Q13: Find the closure operations (Union, Intersection and Complement) for the following two DFAs.



(a)

(b)